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FUZZY RELIABILITY APPRAISAL OF COMPLEX STRUCTURES USING MULTI-VALUED NEUTROSOPHIC SET

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Abstract. One of the most vital areas of reliability engineering is reliability modelling. Here the reliability of a system's components is represented by a multi-valued neutrosophic fuzzy number. In this study, a novel method was developed for estimating fuzzy reliability using multi-valued neutrosophic set theory. This method models uncertainty and ambiguity in real-world circumstances. The suggested approach can be more flexible and intelligently analyse the reliability of fuzzy systems. The suggested method compares several complicated systems with the help of the score function. The score function can aid the decision-maker in making decisions more effectively in decision-making problems.

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1. Introduction

Zadeh [32] was the first who gave the idea of fuzzy set (FS) for handling uncertainty. Since then, the various extensions of FSs have been applied in different realistic problems under uncertain environment. In spite of that, there exist some real-life problems (like when given data is in interval form) that cannot be handled by only using a fuzzy set. This concept of interval-valued fuzzy set (Zadeh, 1975) was used in many techniques for handling data in the form of intervals or lower and upper limits. Atanassov [1] has given the idea of an intuitionistic fuzzy set (IFs) in which both membership and non-membership grades are associated with each element of the given data [2]. But for getting better results in various real-life fields and for handling ambiguous data in multi-criteria decision-making (MCDM) problems, one of the new extensions is the Neutrosophic set (NS), which was introduced by [21], [22], [23], [24], [25], [26]. NS handles such types of data where uncertainty and ambiguity are present in real-world problems. Joshi B.P. et al. [8], [9], [10] utilised different generalisations of FS to tackle MCDM problems.

On the other hand, Neutrosophic set [20] theory plays a key role in the field of management, engineering, and reliability estimation because the ambiguity is present everywhere in many realistic situations. An extension set

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consisting of data in interval form named the interval neutrosophic set (INS) was proposed by Wang et al. [28], [29]. When the operations and comparison techniques of single-valued neutrosophic sets were extended to multiple values, the aforementioned ambiguity still existed in the data set. So Wang et al. [30] introduced the theory of multi-valued neutrosophic sets (MVNS). Subsequently, Peng et al. [19] used the concept of MVNS in the area of decision-making problems, including engineering, management, reliability assessment, etc.

In order to handle fuzzy data, the existing reliability finding techniques fall short. For handling this situation, Chen [5], Pei et al. [18], and Onisawa et al. [16] used fuzzy numbers and their operations for reliability estimation. After that, several researchers proposed different methods related to fuzzy set theory. For instance, Verma et al. [27] introduced a method for dynamic reliability estimation using a special type of fuzzy number as a triangular fuzzy number. Kumar D. et al. [11], [12] used dual hesitant fuzzy set and rough fuzzy set for reliability evaluation. After that, K. Mintu et al. [15] used a Pythagorean fuzzy set for reliability estimation. Jakkula et al. [7] have done the different reliability characteristics of LHD (Lord Haul Dumpers). Das et al. [6] , Kumar et al. [14], and Paramanik et al. [17] have analysed system reliability with various techniques. Later on, Bhadauria et al. [4] and Kumar et al. [13] appraised the reliability of complex units using score and accuracy functions under a fuzzy environment. An approach of reliability optimisation of redundant systems was given by Gao et al. [31] and Azhdari et al [3].

In civil engineering, the reliability of structures (such as bridges, dams, and buildings) often involves uncertain material properties, fluctuating load conditions, and varying environmental factors (e.g., seismic activity, wind loads). A multi-valued neutrosophic set can be used to assess the reliability of these structures by considering three factors:
Truth (T): The degree to which the material or structural component meets the required safety standards.
Indeterminacy (I): The uncertainty or lack of complete knowledge about the system, such as unknown material defects, variations in load conditions, or environmental factors. Falsity (F): The degree to which failure is likely, given the uncertainties.
For example, in the design of a bridge, MVNS can model the reliability of its materials (steel, concrete), environmental factors (rain, earthquakes), and load conditions (traffic), allowing engineers to identify critical points where reinforcement is needed, or failure is likely.

This study discusses the multi-valued neutrosophic set for assessing the reliability of a system as a continuation of previous works. In this research, we propose a novel method for the reliability of various complex structures by using a multi-valued neutrosophic set in Section 2. This discussed technique has been used to appraise the reliability of bridge configuration and their components in Section 3. Here we used the definition of a score function for better comparison between multi-value neutrosophic fuzzy numbers. There are few important illustrations that are considered to justify given invented results in Section 4. In the end, the brief statement is given in the form of a conclusion in Section 5.

2. Preliminaries

Here, some basic information about fuzzy set, neutrosophic set and multi-vauled neutrosophic set has been reviewed.

2.1. **Neutrosophic Set ([21]).** Suppose ξ be the universe. A neutrosophic set (NS) A in ξ is characterized by a truth membership function T_A, a falsity membership function F_A and an indeterminacy membership function I_A. Here T_A, I_A and F_A are real standard elements of [0,1]. It can be written as:

$$A = \{< x, (T_A(x), I_A(x), F_A(x)) >: x \in \xi, T_A, I_A, F_A \in]0, 1[\} \tag{1}$$

where

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3 \tag{2}$$

2.2. **Single valued neutrosophic set ([20]).** Suppose X be a space of points with generic elements in ξ denoted by x. A single valued neutrosophic set A (SVNS) is characterized by truth-membership function $T_A(x)$, a falsity-membership function $F_A(x)$, and an indeterminacy-membership function $I_A(x)$. A SVNS A can be written as

$$A = \{< x, (T_A(x), I_A(x), F_A(x)) >: x \in \xi\} \tag{3}$$

2.3. **Interval valued neutrosophic set ([28]).** Suppose ξ be a space of points with generic elements in X denoted by x. An interval valued neutrosophic set A (IVNS) is characterized by an interval truth-membership function $T_A(x) = [T_A^L, T_A^U]$, an interval falsity-membership function $F_A(x) = [F_A^L, F_A^U]$ and an interval indeterminacy-membership function $I_A(x) = [I_A^L, I_A^U]$, For every point, $x \in \xi, T_A(x), I_A(x), F_A(x) \in [0, 1]$. An IVNS A can be written as

$$A = \{< x, (T_A(x), I_A(x), F_A(x)) >: x \in \xi\} \tag{4}$$

2.4. **Multi valued neutrosophic set ([19]).** Suppose X be a space of points with generic elements in ξ denoted by x, then multi-valued neutrosophic sets A in X is characterized by a truth-membership function $T_A(x)$, a falsity-membership function $F_A(x)$ and a indeterminacy membership function $I_A(x)$. Multi-valued neutrosophic sets can be depicted as

$$A = \{< x, (T_A(x), I_A(x), F_A(x)) >: x \in \xi\} \tag{5}$$

and satisfies the condition
$0 \leq \gamma, \rho, \tau \leq 1, 0 \leq \gamma^+ + \rho^+ + \tau^+ \leq 3, \gamma \; inT_A(x), \rho \in I_A(x), \tau \in F_A(x), \gamma^+ = supT_A(x), \rho^+ = supI_A(x), \tau^+ = supF_A(x)$.

On another words, $A =< T_A, I_A, F_A >$ is called as multi-valued neutrosophic number. If $T_A(x), I_A(x), F_A(x)$ has only one value, the multi-valued neutrosophic sets is called as single valued neutrosophic sets. If $T_A(x) = \phi$, the multi-valued neutrosophic sets is called as double hesitant fuzzy sets. If

$T_A(x) = F_A(x) = \phi$, the multi-valued neutrosophic sets is called as hesitant fuzzy sets.

2.5. **OPERATIONS ON MULTI VALUED NEUTROSOPHIC SET ([19]).** Let $A =< T_A(x), I_A(x), F_A(x) >$ and $B =< T_B(x), I_B(x), F_B(x) >$ are any two MVNNs. The operations for MVNNs are defined as follows.
$(i) A^C =< \cup_{\tau \in F_A}\{\tau\} >$
$(ii) A \oplus B =< \cup_{\gamma_A \in T_A, \gamma_B \in T_B}\{\gamma_A + \gamma_B - \gamma_A\gamma_B\}, \cup_{\rho_A \in I_A, \rho_B \in I_B}\{\rho_A.\rho_B\}, \cup_{\tau_A \in F_A, \tau_B \in F_B}\{\tau_A\tau_B\}$
$(iii) A \oplus B =< \cup_{\gamma_A \in T_A, \gamma_B \in T_B}\{\gamma_A\gamma_B\}, \cup_{\rho_A \in I_A, \rho_B \in I_B}\{\rho_A + \rho_B - \rho_A.\rho_B\}, \cup_{\tau_A \in F_A, \tau_B \in F_B}\{\tau_A + \tau_B - \tau_A\tau_B\}$

2.6. **SCORE FUNCTION OF MULTI VALUED NEUTROSOPHIC NUMBER.** Suppose $A =< T_A, I_A, F_A >$ be an MVNN, then (i) Score function of MVNN A is defined as:

$$S(A) = \frac{1}{l_{T_A}.l_{I_A}.l_{F_A}} \sigma_{\gamma \in T_A, \rho \in I_A, \tau \in F_A}(\gamma_i - \rho_j - \tau_k); \tag{6}$$

(ii) Accuracy function of MVNN A is defined as:

$$H(A) = \frac{1}{l_{T_A}.l_{F_A}} \sigma_{\gamma \in T_A, \tau \in F_A}(\gamma_i - \tau_k); \tag{7}$$

Here, $\gamma_A \in T_A, \rho_A \in I_A, \tau \in F_A$ and $l_{T_A}, l_{I_A}, l_{F_A}$ denote the number of elements in T_A, l_A, F_A respectively.

2.7. **SERIES SYSTEM.** In the series system, the whole system fails only if at least one of the units/ subsystems of the system fails (FIGURE 1).

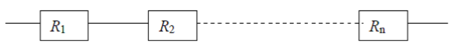

FIGURE 1. Series System

2.8. **PARALLEL SYSTEM.** Parallel system fails only if all the units/ subsystems of the system fails (FIGURE. 2).

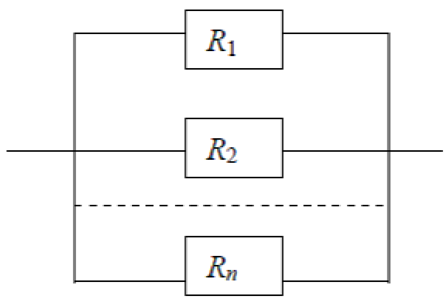

FIGURE 2. Parallel System

2.9. **PARALLEL-SERIES SYSTEM.** The combination where series networks are connected in parallel is called parallel-series system (FIGURE. 3).

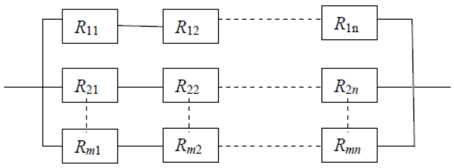

FIGURE 3. Parallel- Series system

2.10. **SERIES-PARALLEL SYSTEM.** The combination where parallel branches are connected in series is known as series-parallel system (FIGURE 4)."

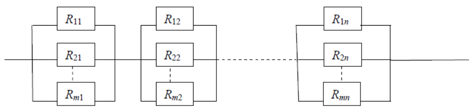

FIGURE 4. Series-Parallel system

3. FUZZY RELIABILITY USING MULTI-VALUED NEUTROSOPHIC FUZZY NUMBER (MVNN)

3.1. **SERIES SYSTEM.** Suppose n components combined in series manner (FIGURE 1) and suppose $R_i = M_i$ be the reliability of the i^th(where $i = 1, 2, 3, ..., n$) component (as a form of MVNN), then the reliability R_s of this combination is appraised as

$R_s = \otimes_{i=1}^{n} R_i = \otimes_{i=1}^{n} M_i$

$R_s = M_1 \otimes M_2 \otimes ... \otimes M_n$

$$R_S = \left\langle \bigcup_{\gamma_i \in T_i} \prod_{i=1}^{n} \gamma_i, \bigcup_{\rho_i \in I_i} \left(\sum_{i=1}^{n} \rho_i - \sum_{i=1, i_1=2}^{n} \rho_i \rho_{i_1} + \sum_{i=1, i_1=2, i_3=3}^{n} \rho_i \rho_{i_1} \rho_{i_2} - \cdots + (-1)^{n+1} \rho_1 \rho_2 \rho_3 \cdots \rho_n \right), \bigcup_{\tau_i \in F_i} \left(\sum_{i=1}^{n} \rho_i - \sum_{i=1, i_1=2}^{n} \tau_i \tau_{i_1} + \sum_{i=1, i_1=2, i_3=3}^{n} \tau_i \tau_{i_1} \tau_{i_2} - \cdots + (-1)^{n+1} \tau_1 \tau_2 \tau_3 \cdots \tau_n \right) \right\rangle$$

3.2. **PARALLEL SYSTEM.** Suppose n components combined in parallel manner (FIGURE 2) and suppose $R_i = M_i$ be the reliability of the i^th (where $i = 1, 2, 3, ..., n$) component (as a form of MVNN), then the reliability R_p of this combination is appraised as

$R_p = \left(\otimes_{i=1}^{n} R_i^c \right)^c = \left(\otimes_{i=1}^{n} M_i^c \right)^c$

$(R_p)^c = (M_1)^c \otimes (M_2)^c \otimes (M_3)^c \otimes ... \otimes (M_n)^c$

$$(R_p)^c = \left\langle \bigcup_{\tau_i \in F_i} \left(\sum_{i=1}^{n} \tau_i - \sum_{i=1,i_1=2}^{n} \tau_i \tau_{i_1} + \sum_{i=1,i_1=2,i_2=3}^{n} \tau_i \tau_{i_1} \tau_{i_2} \right.\right.$$
$$\left.\left. - \cdots + (-1)^{n+1} \tau_1 \tau_2 \tau_3 \cdots \tau_n \right), \bigcup_{\rho_i \in I_i} \prod_{i=1}^{n} \rho_i, \bigcup_{\gamma_i \in T_i} \prod_{i=1}^{n} \gamma_i \right\rangle$$

3.3. **PARALLEL-SERIES SYSTEM.** In FIGURE 3, suppose R_i^j be the reliability of the j^{th} unit of the i^th branch (where $j = 1$ to m and $i = 1$ to n), then the fuzzy reliability of the parallel-series system is appraised as

$$R_{ps}^c = \otimes_{i=1}^{n} \left(\otimes_{j=1}^{m} R_i^j \right)^c$$
$$R_{ps}^c = \otimes_{i=1}^{n} \left(\otimes_{j=1}^{m} M_i^j \right)^c$$
$$(R_{ps})^c = \otimes_{i=1}^{n} \left[M_i^1 \otimes M_i^2 \otimes \cdots \otimes M_i^m \right]^c$$
$$(R_{ps})^c = \otimes_{i=1}^{n} \left[(h_i^1) \otimes (h_i^2) \otimes \cdots \otimes (h_i^m) \right]^c$$
$$R_{ps}^c = \bigotimes_{i=1}^{n} \left\{ \bigcup_{\gamma_i^j \in T_i^j, \rho_i^j \in I_i^j, \tau_i^j \in F_i^j, (i=1,2,\ldots n),(j=1,2,\ldots m)} \left(\gamma_i^1, \gamma_i^2, \gamma_i^3, \ldots, \gamma_i^m \right), \right.$$
$$\left(\sum_{j=1}^{m} \rho_i^j - \sum_{j,j_1=1}^{m} \rho_i^j \rho_i^{j_1} + \cdots + (-1)^{m+1} \rho_i^1 \rho_i^2 \ldots \rho_i^m \right)$$
$$\left. \left(\sum_{j=1}^{m} \tau_i^j - \sum_{j,j_1=1}^{m} \tau_i^j \tau_i^{j_1} + \cdots + (-1)^{m+1} \tau_i^1 \tau_i^2 \ldots \tau_i^m \right) \right\}^c$$
$$R_{ps}^c = \bigotimes_{i=1}^{n} \left[\bigcup_{\gamma_i^j \in T_i^j, \rho_i^j \in I_i^j, \tau_i^j \in F_i^j, (i=1,2,\ldots n),(j=1,2,\ldots m)} \left(\sum_{j=1}^{m} \tau_i^j - \sum_{j,j_1=1}^{m} \tau_i^j \tau_i^{j_1} + \right.\right.$$
$$\left. \cdots + (-1)^{m+1} \tau_i^1 \tau_i^2 \ldots \tau_i^m), (\gamma_i^1, \gamma_i^2, \ldots, \gamma_i^m), (\rho_i^1 \rho_i^2 \ldots \rho_i^m) \right]$$
$$R_{ps}^c = \bigcup_{\gamma_i^j \in T_i^j, \rho_i^j \in I_i^j, \tau_i^j \in F_i^j, (i=1,2,\ldots n),(j=1,2,\ldots m)} \left\{ \left(\sum_{j=1}^{m} \tau_1^j - \sum_{j,j_1=1}^{m} \tau_1^j \tau_1^{j_1} + \cdots + (-1)^{m+1} \tau_1^1 \tau_1^2 \ldots \tau_1^m \right) \right.$$
$$\left(\sum_{j=1}^{m} \tau_2^j - \sum_{j,j_1=1}^{m} \tau_2^j \tau_2^{j_1} + \cdots + (-1)^{m+1} \tau_2^1 \tau_2^2 \ldots \tau_2^m \right).$$
$$\left. \left(\sum_{j=1}^{m} \tau_n^j - \sum_{j,j_1=1}^{m} \tau_n^j \tau_n^{j_1} + \cdots + (-1)^{m+1} \tau_n^1 \tau_n^2 \ldots \tau_n^m \right) \right\},$$
$$\left\{ \sum_{j=1}^{m} (\rho_i^1 \rho_i^2 \ldots \rho_i^m) - \sum_{i,i_1=1}^{n} (\rho_i^1 \rho_i^2 \ldots \rho_i^m).(\rho_{i_1}^1 \rho_{i_1}^2 \ldots \rho_{i_1}^m) \right.$$
$$\left. + \ldots (-1)^{n+1} (\rho_1^1 \rho_1^2 \ldots \rho_1^m)(\rho_2^1 \rho_2^2 \ldots \rho_2^m) \ldots (\rho_n^1 \rho_n^2 \ldots \rho_n^m) \right\},$$
$$\left\{ \sum_{i=1}^{n} (\gamma_i^1 \gamma_i^2 \ldots \gamma_i^m) - \sum_{i,i_1=1}^{n} (\gamma_i^1 \gamma_i^2 \ldots \gamma_i^m).(\gamma_{i_1}^1 \gamma_{i_1}^2 \ldots \gamma_{i_1}^m) \right.$$
$$\left. + \ldots (-1)^{n+1} (\gamma_1^1 \gamma_1^2 \ldots \gamma_1^m)(\gamma_2^1 \gamma_2^2 \ldots \gamma_2^m) \ldots (\gamma_n^1 \gamma_n^2 \ldots \gamma_n^m) \right\}$$
$$R_{ps} = \bigcup_{\gamma_i^j \in T_i^j, \rho_i^j \in I_i^j, \tau_i^j \in F_i^j, (i=1,2,\ldots n),(j=1,2,\ldots m)} \left[\left\{ \sum_{i=1}^{n} (\gamma_i^1 \gamma_i^2 \ldots \gamma_i^m) \right.\right.$$
$$- \sum_{i,i_1=1}^{n} (\gamma_i^1 \gamma_i^2 \ldots \gamma_i^m).(\gamma_{i_1}^1 \gamma_{i_1}^2 \ldots \gamma_{i_1}^m) + \ldots$$
$$\left. (-1)^{n+1} (\gamma_1^1 \gamma_1^2 \ldots \gamma_1^m)(\gamma_2^1 \gamma_2^2 \ldots \gamma_2^m) \ldots (\gamma_n^1 \gamma_n^2 \ldots \gamma_n^m) \right\},$$

$$\Big\{ \sum_{i=1}^{n}(\rho_i^1\rho_i^2\dots\rho_i^m) - \sum_{i,i_1=1}^{n}(\rho_i^1\rho_i^2\dots\rho_i^m).(\rho_{i_1}^1\rho_{i_1}^2\dots\rho_{i_1}^m)$$
$$+\dots(-1)^{n+1}(\rho_1^1\rho_1^2\dots\rho_1^m)(\rho_2^1\rho_2^2\dots\rho_2^m)\dots(\rho_n^1\rho_n^2\dots\rho_n^m)\Big\},$$
$$\Big\{ \Big(\sum_{j=1}^{m}\tau_1^j - \sum_{j,j_1=1}^{m}\tau_1^j\tau_1^{j_1} + \dots + (-1)^{m+1}\tau_1^1\tau_1^2\dots\tau_1^m\Big)$$
$$\Big(\sum_{j=1}^{m}\tau_2^j - \sum_{j,j_1=1}^{m}\tau_2^j\tau_2^{j_1} + \dots + (-1)^{m+1}\tau_2^1\tau_2^2\dots\tau_2^m\Big)$$
$$\Big(\sum_{j=1}^{m}\tau_n^j - \sum_{j,j_1=1}^{m}\tau_n^j\tau_n^{j_1} + \dots + (-1)^{m+1}\tau_n^1\tau_n^2\dots\tau_n^m\Big)\Big\}\Big]$$

3.4. **SERIES -PARALLEL SYSTEM.** In figure 4, suppose R_j^i be the reliability of the i^{th} unit of j^{th} branch (where $j = 1$ to m and $i = 1$ to n), then the fuzzy reliability of the series -parallel system is appraised as

$$R_{sp} = \bigotimes_{j=1}^{m}\Big[\bigotimes_{i=1}^{n} R_i^j\Big]^c$$
$$R_{sp} = \bigotimes_{j=1}^{m}\Big[\bigotimes_{i=1}^{n} M_i^j\Big]^c$$
$$R_{sp} = \otimes_{j=1}^{m}\Big[M_1^j \otimes M_2^j \otimes \dots \otimes M_n^j\Big]^c$$
$$R_{sp} = \otimes_{i=1}^{n}\Big[h_i^1 \otimes h_i^2 \otimes \dots \otimes h_i^m\Big]^c$$
$$R_{sp} = \bigotimes_{i=1}^{n}\Big[\bigcup_{\gamma_i^j\in T_i^j,\rho_i^j\in I_i^j,\tau_i^j\in F_i^j,(i=1,2,\dots n),(j=1,2,\dots m)}\Big\{\Big(\sum_{i=1}^{n}\tau_i^j - \sum_{i,i_1=1}^{n}\tau_i^j\tau_{i_1}^j +$$
$$\dots + (-1)^{n+1}\tau_1^j\tau_2^j\dots\tau_m^j\Big),(\gamma_i^1,\gamma_i^2,\dots,\gamma_i^m),(\rho_i^1\rho_i^2\dots\rho_i^m)\Big\}\Big]$$
$$R_{sp} = \bigcup_{\gamma_i^j\in T_i^j,\rho_i^j\in I_i^j,\tau_i^j\in F_i^j,(i=1,2,\dots n),(j=1,2,\dots m)}\Big[\Big\{\Big(\sum_{i=1}^{n}\tau_i^1 - \sum_{i,i_1=1}^{n}\tau_i^1\tau_{i_1}^1 + \dots + (-1)^{n+1}\tau_1^1\tau_i^1\dots\tau_i^1\Big)$$
$$\Big(\sum_{i=1}^{n}\tau_i^2 - \sum_{i,i_1=1}^{n}\tau_i^2\tau_{i_1}^2 + \dots + (-1)^{n+1}\tau_i^2\tau_i^2\dots\tau_i^2\Big)$$
$$\Big(\sum_{i=1}^{n}\tau_i^n - \sum_{i,i_1=1}^{n}\tau_i^n\tau_{i_1}^n + \dots + (-1)^{n+1}\tau_i^n\tau_i^n\dots\tau_i^n\Big)\Big\},$$
$$\Big\{ \sum_{j=1}^{n}(\rho_1^j\rho_2^j\dots\rho_n^j) - \sum_{j,j_1=1}^{n}(\rho_1^j\rho_2^j\dots\rho_n^j).(\rho_1^{j_1}\rho_2^{j_1}\dots\rho_n^{j_1})$$
$$+\dots(-1)^{n+1}(\rho_1^1\rho_2^1\dots\rho_n^1)(\rho_1^2\rho_2^2\dots\rho_n^2)\dots(\rho_1^m\rho_2^m\dots\rho_n^m)\Big\},$$
$$\Big\{ \sum_{j=1}^{n}(\gamma_1^j\gamma_2^j\dots\gamma_n^j) - \sum_{j,j_1=1}^{n}(\gamma_1^j\gamma_2^j\dots\gamma_n^j).(\gamma_1^{j_1}\gamma_2^{j_1}\dots\gamma_n^{j_1})$$
$$+\dots(-1)^{n+1}(\gamma_1^1\gamma_2^1\dots\gamma_n^1)(\gamma_1^2\gamma_2^2\dots\gamma_n^2)\dots(\gamma_1^m\gamma_2^m\dots\gamma_n^m)\Big\}\Big]$$

4. Example

Suppose decision maker provide reliabilities of three components (as form of MVNN) given as:
$R_1 = M_1 = \{(0.1), (0.2, 0.11), (0.3)\}$
$R_2 = M_2 = \{(0.5), (0.6), (0.4, 0.1)\}$
$R_3 = M_3 = \{(0.2), (0.8), (0.1)\}$
Now, reliability of various structures is appraised as follows.

4.1. **Series System.** Suppose R_1, R_2 and R_3 are connected in series manner (FIGURE 1). So, fuzzy reliability R_s of this system is appraised by
$R_s = \otimes_{i=1}^{3} R_i = \otimes_{i=1}^{3} M_i$
$R_s = \{(0.01), (0.936, 0.81), (0.622, 0.433)\}$
So, by using (6) & (7)
S(R_s) = -0.4635

4.2. **Parallel System.** Suppose R_1, R_2 and R_3 are connected in parallel manner (FIGURE 2). Therefore, fuzzy reliability R_p of the system is appraised by

$$R_p = \left(\otimes_{i=1}^{3} R_i^c \right)^c = \left(\otimes_{i=1}^{3} M_i^c \right)^c$$

$R_P = \{(0.64), (0.096, 0.053), (0.012, 0.03)\}$
So, by using (6) & (7)
S(R_P) = 0.18125

4.3. **Parallel-series system.** Suppose R_1, R_2 and R_3 are connected in a series manner to make single branch and these types of three branches are connected in parallel manner (FIGURE 3). So, fuzzy reliability R_{PS} is appraised by
$R_{PS} = \{(0.28), (0.82, 0.71, 0.71, 0.61, 0.71, 0.61, 0.81, 0.53),$
$(0.240, 0.167, 0.167, 0.116, 0.167, 0.116, 0.116, 0.081)\}$
So, by using (6) & (7)
S(R_{PS}) = -0.183125

4.4. **Series-parallel system.** Suppose R_1, R_2 and R_3 are connected in a parallel manner to make single branch and these types of three branches are connected in series manner (FIGURE 4). So, fuzzy reliability R_{SP} is appraised by
$R_{SP} = \{(0.262), (0.262, 0.226, 0.226, 0.189, 0.226, 0.189, 0.189, 0.151),$
$(0.035, 0.053, 0.053, 0.071, 0.053, 0.071, 0.071, 0.087)\}$
So, by using (6) & (7)
S(R_{SP})=-0.0522937
By comparing all above results, it is clear that reliability is superior in parallel structure (See FIGURE 5).
$S(R_p) > S(R_{SP}) > S(R_{PS}) > S(R_s) \implies R_p > R_{SP} > R_{PS} > R_s$

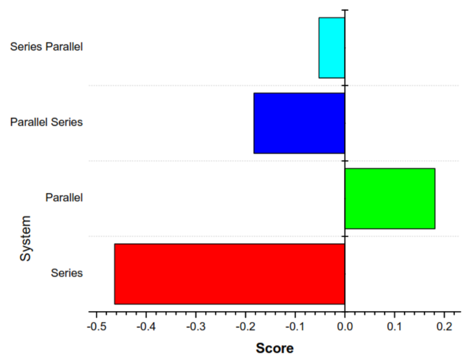

FIGURE 5. Comparison of results

5. FAULT TREE ANALYSIS FOR THE RELIABILITY OF FIRE ALARM SYSTEM USING MULTI- VALUED NEUTROSOPHIC FUZZY NUMBER

To demonstrate the proposed method for analysing the reliability of a fire alarm system in a building under fuzzy environment, two models have been taken which consists of smoke detector, wiring, alarm and power supply.
MODEL I
The fire alarm system in a building under fuzzy environment consists of four components i.e., the smoke detector, wire, alarm, and power supply that are connected in series. The effectiveness of each component determines the overall dependability of the system. The entire fire alarm system will not work if any one of its parts fails. In order to offer backup illumination in the event of a power loss, the power supply is made up of two batteries, Battery 1 and Battery 2, which are connected in parallel. Reliability is ensured because the lights will continue to run on the other battery even if one fails (FIGURE 6).

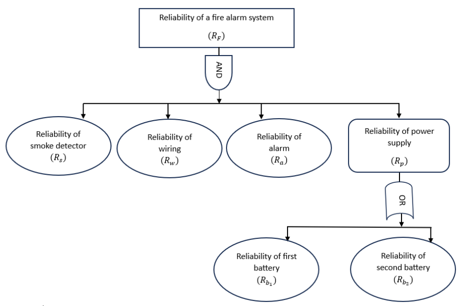

FIGURE 6. Fault tree for fire alarm system: MODEL I

Let us consider the possible reliabilities in the form of multi-valued neutrosophic numbers such as $R_s, R_w, R_a, R_{b_1}, R_{b_2}$ for smoke detector, wiring, alarm, first and second battery respectively are as follows:

$R_s = \{0.1, \{0.2, 0.4\}, 0.5\}$
$R_w = \{\{0.2, 0.3, 0.4\}, 0.2, 0.4\}$
$R_a = \{0.5, 0.6, \{0.2, 0.3\}\}$
$R_{b_1} = \{\{0.1, 0.2\}, 0.4, 0.6\}$
$R_{b_2} = \{0.3, \{0.2, 0.4\}, 0.5\}$

Then, the reliability for the top event i.e., fire alarm system of model I can be calculated as:
$R_F = R_s \otimes R_w \otimes R_a \otimes R_p$
$R_F = R_s \otimes R_w \otimes R_a \otimes (R_{b_1}^C \otimes R_{b_2}^C)^C$
$R_F = \{\{0.0037, 0.0044.0.00555, 0.0066, 0.074, 0.088\}, \{0.76448, 0.78496, 0.82336, 0.83872\}, \{0.832, 0.853\}\}$
So, by using (6) & (7)
$S(R_F) = -1.6.$

MODEL II

Model II is identical to Model I, however the power supply has four batteries instead of two that are connected in parallel manner (FIGURE 7).

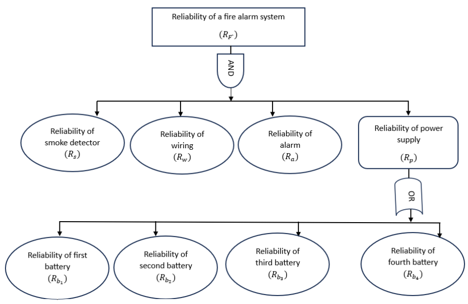

FIGURE 7. Fault tree for fire alarm system: MODEL II

Let us consider the possible reliabilities in the form of multi-valued neutrosophic numbers such as$R_s, R_w, R_a, R_{b_1}, R_{b_2}, R_{b_3}$ and R_{b_4} for smoke detector, wiring, alarm, first, second, third and fourth battery respectively are as follows:
$R_s = \{0.1, \{0.2, 0.4\}, 0.5\}$
$R_w = \{\{0.2, 0.3, 0.4\}, 0.2, 0.4\}$
$R_a = \{0.5, 0.6, \{0.2, 0.3\}\}$
$R_{b_1} = \{\{0.1, 0.2\}, 0.4, 0.6\}$
$R_{b_2} = \{0.3, \{0.2, 0.4\}, 0.5\}$
$R_{b_3} = \{0.1, \{0.2, 0.3\}, 0.5\}$
$R_{b_4} = \{0.6, 0.1, 0.2\}$

Then, the reliability for the top event i.e., fire alarm system of model II can be calculated as:

$R_{F'} = R_s \otimes R_w \otimes R_a \otimes R_p$
$= R_s \otimes R_w \otimes R_a \otimes (R_{b_1}^C \otimes R_{b_2}^C \otimes R_{b_3}^C \otimes R_{b_4}^C)^C$
$= \{\{0.007732, 0.007984, 0.011598, 0.011976, 0.15464, 0.15968\}, \{0.7444096,$
$0.7446144, 0.7448192, 0.7452288, 0.8083072, @0.8084608, 0.8086144, 0.8089216\},$
$\{0.7672, 0.7963\}\}$
So, by using (6) & (7)
$S(R_{F'}) = -1.4$

Therefore, by comparing the score function for two models, it can be easily concluded that the reliability of Model II is greater than that of Model I as Model II consists of four batteries instead of two batteries as in model I.

6. CONCLUSION

Here, a new technique is proposed for appraising fuzzy reliability of given complex structure under multi-valued neutrosophic fuzzy environment. It is obtained from the results and the graph given in FIGURE 5 that the reliability is superior in parallel configuration which follow the basic fact that reliability of parallel system always greater than series system. It is also verified in section 5, where we have compared two models in which fuzzy reliability of model II is better than model I. Here we have used advanced comparison method (using score function) to make a better comparison among multi-valued neutrosophic fuzzy numbers. Finally, this new technique helps for appraising reliability in many situations where vagueness, uncertainty and ambiguity are present in provided data. In future, this method can be used in many extensions of fuzzy set for reliability evaluation.

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